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Worldline approach to vector and antisymmetric tensor fields II

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Abstract: We extend the worldline description of vector and antisymmetric tensor fields coupled to gravity to the massive case. In particular, we derive a worldline path integral representation for the one-loop effective action of a massive antisymmetric tensor field of rank p (a massive p-form) whose dynamics is dictated by a standard Proca-like lagrangian coupled to a background metric. This effective action can be computed in a proper time expansion to obtain the corresponding Seeley-DeWitt coefficients a_0, a_1, a_2. The worldline approach immediately shows that these coefficients are derived from the massless ones by the simple shift $D \to D + 1$, where D is the spacetime dimension. Also, the worldline representation makes it simple to derive exact duality relations. Finally, we use such a representation to calculate the one-loop contribution to the graviton self-energy due to both massless and massive antisymmetric tensor fields of arbitrary rank, generalizing results already known for the massless spin 1 field (the photon).

Keywords: Gauge Symmetry, Duality in Gauge Field Theories, Sigma Models.

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Contents

1. Introduction

In a previous paper [1] we have studied a worldline approach to massless antisymmetric tensor fields of arbitrary rank. In the present paper we extend such an approach to the massive case and use it: (i) to identify the Seeley-DeWitt coefficients a_0, a_1 and a_2 for massive antisymmetric tensor fields of arbitrary rank in arbitrary dimensions, (ii) to study duality relations, and (iii) to compute the one-loop contribution to the graviton self-energy (or equivalently, the stress tensor two-point function) due to both massless and massive antisymmetric tensors. This latter result generalizes the calculation for the massless spin 1 field (the photon) [2] to the case of massless and massive antisymmetric tensor fields of arbitrary rank.

We denote our previous work [1] as paper I, and refer to that paper for a general introduction to the subject. Thus, in section 2 we directly discuss the $N = 2$ spinning particle model appropriate for describing massive antisymmetric tensor fields in first quantization. In section 3 we construct the corresponding path integral on the circle (one-dimensional torus) to obtain the worldline representation of the effective action of a massive p-form. In this section we show that the Seeley-DeWitt coefficients for massive antisymmetric tensors are related to the ones for massless tensors by the simple shift $D \to D+1$, where D is the spacetime dimension. Thus we can readily compute them. We also discuss duality relations between massive p- and $(D-p-1)$-forms and, at the level of unregulated effective actions, find that there is a mismatch proportional to the integral of the Euler topological density of target spacetime. In section 4 we extract the 2-point function from the effective action. It describes the 1PI one-loop graphs contributing to the graviton self-energy. This generalizes the worldline calculations for scalars and Dirac fermions ($N=0$ and $N=1$ spinning particles) [8, 9] to the case of massless and massive antisymmetric tensors of arbitrary rank ($N=2$ spinning particle). As a special case we recover the result for the massless photon computed in [2]. In section 5 we point out a relation of the worldline representation of the effective action with the QFT description of massive tensors given in terms of a gauge invariant lagrangian with Stückelberg fields. Then we briefly state our conclusions. We use the appendix to explicitly list the Seeley-DeWitt coefficients a_0, a_1 and a_2 for massive p-forms with $p=0,1,2,3,4,5$ in arbitrary D dimensions, and to describe in some details the calculation of the two-point function.

2. Massive $N=2$ spinning particle action

The $N=2$ spinning particle can be used to describe massless and massive antisymmetric tensors in first quantization [3–7]. In paper I we reviewed the massless case. The inclusion of a mass term is straightforward, and can be obtained by considering the massless case in one dimension higher. In fact, one can use the massless spinning particle in $D+1$ dimensions, and then eliminate one dimension (for example x^5) by setting $p_5 = m$. The coordinate x^5 can be dropped from the action (it appears only as a total derivative), while the corresponding $N=2$ fermionic partners are retained and denoted by θ and $\bar\theta$. This procedure gives the massive $N=2$ action, which in phase space reads

$$S = \int dt \big[p_\mu \dot x^\mu + i\bar\psi_\mu \dot\psi^\mu + i\bar\theta\dot\theta - eH - i\bar\chi Q - i\chi\bar Q - aJ + qa\big] \tag{2.1}$$

where the constraints $C \equiv (H, Q, \bar Q, J)$ are given by

$$H = \frac{1}{2}(p_\mu p^\mu + m^2) \qquad Q = p_\mu \psi^\mu + m\theta \qquad \bar Q = p_\mu \bar\psi^\mu + m\bar\theta \qquad J = \bar\psi^\mu \psi_\mu + \bar\theta\theta\,. \tag{2.2}$$

They satisfy the Poisson brackets algebra (the $N=2$ algebra)

$$\{Q, \bar Q\}_{PB} = -2iH\,, \qquad \{J, Q\}_{PB} = iQ\,, \qquad \{J, \bar Q\}_{PB} = -i\bar Q \tag{2.3}$$

and are gauged by the gauge fields $G \equiv (e, \bar\chi, \chi, a)$. The quantized Chern-Simons coupling $q \equiv \frac{D+1}{2} - p - 1$ allows to describe an antisymmetric tensor of rank p [7]. This is immediately seen in canonical quantization. The phase space variables are turned into operators satisfying the (anti)commutation relations (we use $\hbar = 1$)

$$[\hat x^\mu, \hat p_\nu] = i\delta^\mu_\nu\,, \qquad \{\hat\psi^\mu, \hat\psi^\dagger_\nu\} = \delta^\mu_\nu\,, \qquad \{\hat\theta, \hat\theta^\dagger\} = 1\,. \tag{2.4}$$

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States of the Hilbert space can be described by functions of the coordinates x^μ, ψ^μ, θ (more precisely, one uses coherent states for the fermions), and since ψ^μ and θ are Grassmann variables, the wave function has the following general expansion

$$\begin{aligned}\phi(x,\psi,\theta) &= F(x) + F_\mu(x)\psi^\mu + \frac{1}{2}F_{\mu_1\mu_2}(x)\psi^{\mu_1}\psi^{\mu_2} + \ldots + \frac{1}{D!}F_{\mu_1\ldots\mu_D}(x)\psi^{\mu_1}\cdots\psi^{\mu_D} + \\ &\quad + im\bigg(A(x)\theta + A_\mu(x)\theta\psi^\mu + \frac{1}{2}A_{\mu_1\mu_2}(x)\theta\psi^{\mu_1}\psi^{\mu_2} + \cdots + \\ &\qquad + \frac{1}{D!}A_{\mu_1\ldots\mu_D}(x)\theta\psi^{\mu_1}\cdots\psi^{\mu_D}\bigg). \end{aligned} \tag{2.5}$$

The imaginary unit i makes it possible to impose reality conditions on the fields and the factor m is introduced for obtaining a standard normalization of the A fields.

The classical constraints C become operators $\hat{C}$ which are used to select the physical states through the requirements $\hat{C}|\phi_{phys}\rangle = 0$, valid for all constraints except $\hat{J}$ which gives $\hat{J}|\phi_{phys}\rangle = q|\phi_{phys}\rangle$ because of the Chern-Simons coupling. In the above coordinate representation these constraints take the form of differential operators

$$\begin{aligned}\hat{H} &= \frac{1}{2}(-\partial_\mu\partial^\mu + m^2) \qquad \hat{Q} = -i\psi^\mu\partial_\mu + m\theta \qquad \hat{Q}^\dagger = -i\partial_\mu\frac{\partial}{\partial\psi_\mu} + m\frac{\partial}{\partial\theta} \\ \hat{J} &= -\frac{1}{2}\Big[\psi^\mu, \frac{\partial}{\partial\psi^\mu}\Big] - \frac{1}{2}\Big[\theta, \frac{\partial}{\partial\theta}\Big] = -\psi^\mu\frac{\partial}{\partial\psi^\mu} - \theta\frac{\partial}{\partial\theta} + \frac{D+1}{2}\end{aligned} \tag{2.6}$$

where in $\hat{J}$ we have antisymmetrized $\hat{\psi}^\mu$, $\hat{\psi}^\dagger_\mu$ and $\hat{\theta}$, $\hat{\theta}^\dagger$ to resolve an ordering ambiguity. The $\hat{J} = q$ constraint selects states with only $p+1$ Grassmann variables, namely

$$\phi_{phys}(x,\psi) = \frac{1}{(p+1)!}F_{\mu_1\ldots\mu_{p+1}}(x)\psi^{\mu_1}\cdots\psi^{\mu_{p+1}} + \frac{im}{p!}A_{\mu_1\ldots\mu_p}(x)\theta\psi^{\mu_1}\cdots\psi^{\mu_p}\,. \tag{2.7}$$

The constraints $\hat{Q}|\phi_{phys}\rangle = 0$ gives the Bianchi identities for F_{p+1} and solves them in terms of A_p

$$\partial_{[\mu}F_{\mu_1\mu_2\ldots\mu_{p+1}]} = 0\,, \qquad F_{\mu_1\mu_2\ldots\mu_{p+1}} = \partial_{[\mu_1}A_{\mu_2\ldots\mu_{p+1}]} \tag{2.8}$$

while the constraint $\hat{Q}^\dagger|\phi_{phys}\rangle = 0$ produces the Proca equations together with the familiar longitudinal constraint on A_p

$$\partial^{\mu_1}F_{\mu_1\ldots\mu_{p+1}} = m^2 A_{\mu_2\ldots\mu_{p+1}}\,, \qquad \partial^{\mu_1}A_{\mu_1\ldots\mu_p} = 0\,. \tag{2.9}$$

The constraint $\hat{H}|\phi_{phys}\rangle = 0$ is automatically satisfied as a consequence of the algebra $\{\hat{Q}, \hat{Q}^\dagger\} = 2\hat{H}$.

As these constraints reproduce the Proca field equations (Proca derived them for a massive photon, but the generalization to an arbitrary p-form is immediate), one may conclude that the massive $N = 2$ spinning particle describes a massive antisymmetric tensor field with the standard Proca action

$$S_p^{QFT}[A_p] = \int d^Dx\left[-\frac{1}{2(p+1)!}F^2_{\mu_1\ldots\mu_{p+1}} - \frac{m^2}{2p!}A^2_{\mu_1\ldots\mu_p}\right]. \tag{2.10}$$

We now couple the massive spinning particle model to the target space metric, go to configuration space, perform a Wick rotation to euclidean time ($t \to -i\tau$, and $a \to ia$ to keep the gauge group U(1) compact) to obtain the euclidean action S_E that should be quantized for describing in first quantization a massive antisymmetric tensor field of rank p coupled to gravity. This action reads

$$\begin{aligned} S_E[\tilde{X}, G; g_{\mu\nu}] = \int_0^1 d\tau \Big[& \frac{1}{2} e^{-1} g_{\mu\nu} (\dot{x}^\mu - \bar{\chi}\psi^\mu - \chi\bar{\psi}^\mu)(\dot{x}^\nu - \bar{\chi}\psi^\nu - \chi\bar{\psi}^\nu) + \\ & + \bar{\psi}_a (\dot{\psi}^a + ia\psi^a + \dot{x}^\mu {\omega_\mu}^{ab} \psi_b) - \frac{e}{2} R_{abcd} \bar{\psi}^a \psi^b \bar{\psi}^c \psi^d - iqa + \\ & + \bar{\theta}(\dot{\theta} + ia\theta) + im(\bar{\chi}\theta + \chi\bar{\theta}) + \frac{1}{2} e m^2 \Big] \end{aligned} \tag{2.11}$$

where $\tilde{X} \equiv (x^\mu, \psi^a, \bar{\psi}^a, \theta, \bar{\theta})$ and $G \equiv (e, \bar{\chi}, \chi, a)$. We recall that the parameter q is quantized as

$$q = \frac{D+1}{2} - p - 1 \tag{2.12}$$

in order to describe a massive p-form. The gauge symmetries of the gravity multiplet on the worldline are unchanged (they do not depend on the target space geometry)

$$\begin{aligned} \delta e &= \dot{\xi} + 2\bar{\chi}\epsilon + 2\chi\bar{\epsilon} \\ \delta\chi &= \dot{\epsilon} + ia\epsilon - i\alpha\chi \\ \delta\bar{\chi} &= \dot{\bar{\epsilon}} - ia\bar{\epsilon} + i\alpha\bar{\chi} \\ \delta a &= \dot{\alpha} \end{aligned} \tag{2.13}$$

and need to be gauge fixed to perform quantum calculations.

3. Quantization on a torus

The quantization on the one-dimensional torus goes through as in paper I, to which we refer for details. Choosing the gauge $\hat{G} = (\beta, 0, 0, \phi)$ for the supergravity multiplet, and inserting the factor $-1/2$ for correct overall normalization gives the following worldline representation for the one-loop effective action of a massive p-form

$$\Gamma_p^{QFT}[g_{\mu\nu}] = -\frac{1}{2} \int_0^\infty \frac{d\beta}{\beta} \int_0^{2\pi} \frac{d\phi}{2\pi} \Big(2\cos\frac{\phi}{2}\Big)^{-2} \int_{T^1} \mathcal{D}\tilde{X}\, e^{-S_E[\tilde{X}, \hat{G}; g_{\mu\nu}]} \tag{3.1}$$

where T^1 denotes the one-dimensional torus (a circle), and fermions have antiperiodic boundary conditions around the torus.

To present this effective action in a more explicit form, we integrate over θ and $\bar{\theta}$ which are free and give rise to an extra factor of the fermionic determinant $\det(\partial_\tau + i\phi) = (2\cos\frac{\phi}{2})$. Then we extract explicitly the normalization of the path integral measure together with the bosonic zero modes x_0^μ to obtain

$$\Gamma_p^{QFT}[g_{\mu\nu}] = -\frac{1}{2} \int_0^\infty \frac{d\beta}{\beta} e^{-\frac{1}{2}m^2\beta} \int_0^{2\pi} \frac{d\phi}{2\pi} \Big(2\cos\frac{\phi}{2}\Big)^{D-1} e^{iq\phi} \int \frac{d^D x_0 \sqrt{g(x_0)}}{(2\pi\beta)^{\frac{D}{2}}} \big\langle e^{-S_{\rm int}} \big\rangle \,. \tag{3.2}$$

The expectation values are normalized to one (i.e. $\langle 1\rangle = 1$, with propagators given in paper I), and the interactions are obtained from the nonlinear $N = 2$ sigma model

$$S[X,\hat{G};g_{\mu\nu}] = \frac{1}{\beta}\int_0^1 d\tau \left[\frac{1}{2}g_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu + \bar{\psi}_a(\dot{\psi}^a + i\phi\psi^a + \dot{x}^\mu\omega_\mu{}^a{}_b\psi^b) - \frac{1}{2}R_{abcd}\bar{\psi}^a\psi^b\bar{\psi}^c\psi^d\right] \tag{3.3}$$

which is the same as the one needed in the massless case (by X we denote the fields $(x^\mu, \psi^a, \bar{\psi}^a)$). The only difference from the massless case is the presence of the mass term $e^{-\frac{1}{2}m^2\beta}$ in the first integral of (3.2) and the replacement $D \to D+1$ which appears in the second integral of (3.2). This shows that it is quite simple to obtain the Seeley-DeWitt coefficients in the massive case using our previous results for the massless case. It is enough to shift $D \to D+1$ in the result for the "massless" coefficients given in paper I to obtain the "massive" ones. We list for convenience the explicit coefficients for the case $p = 0,1,2,3,4,5$ in appendix A. The general case is obtained by using the general formulas of paper I and shifting $D \to D+1$.

Let us now discuss duality relations. To this end it is useful to recast (3.2) using the Wilson loop variable $w = e^{i\phi}$ and using an operatorial language

$$\begin{aligned}\Gamma_p^{QFT}[g_{\mu\nu}] &= -\frac{1}{2}\int_0^\infty \frac{d\beta}{\beta}e^{-\frac{1}{2}m^2\beta}\oint_\gamma \frac{dw}{2\pi i w}\frac{w}{(1+w)^2}\,\mathrm{Tr}\left[w^{\hat{N}-p-1}e^{-\beta\hat{H}}\right]\\ &\equiv -\frac{1}{2}\int_0^\infty \frac{d\beta}{\beta}e^{-\frac{1}{2}m^2\beta}Z_p(\beta)\,.\end{aligned} \tag{3.4}$$

Here the path integral on the torus has been represented by the trace in the matter sector $(\tilde{X})$ of the Hilbert space of the spinning particle, $\hat{N} = \hat{\psi}^\mu\hat{\psi}^\dagger_\mu + \hat{\theta}\hat{\theta}^\dagger$ is the (anti) fermion number operator, and $\hat{H}$ is the quantum hamiltonian without the coupling to the gauge field, which has been explicitly factorized. As discussed in paper I, the regulated contour of integration for the U(1) modular parameter w is the unit circle γ with the point $w = -1$ excluded, see figure 1.

One expects a massive p-form to be dual to a massive $(D-p-1)$-form. Indeed changing $p \to (D-p-1)$ in (3.2) only changes the Chern-Simons coupling $q \to -q$. This can be undone by a change of integration variable $\phi \to \phi' = -\phi$, or equivalently $w \to w' = \frac{1}{w}$ in (3.4), to go back to the original expression, except that now the new contour γ' is changed to include the pole $w' = -1$, see figure 2.

The mismatch Δ is easily computed as the residue at this pole

$$\begin{aligned}\Delta &= \mathrm{Res}\left(\frac{1}{(1+w)^2}\,\mathrm{Tr}\left[w^{\hat{N}-p-1}e^{-\beta\hat{H}}\right], w=-1\right)\\ &= \frac{d}{dw}\,\mathrm{Tr}\left[w^{\hat{N}-p-1}e^{-\beta\hat{H}}\right]\Big|_{w=-1}\\ &= \mathrm{Tr}\left[\big(\hat{N}-p-1\big)(-1)^{\hat{N}-p-2}e^{-\beta\hat{H}}\right]\\ &= (-1)^p\,\mathrm{Tr}\left[\hat{N}(-1)^{\hat{N}}e^{-\beta\hat{H}}\right] + (-1)^{p+1}(p+1)\,\mathrm{Tr}\left[(-1)^{\hat{N}}e^{-\beta\hat{H}}\right]\\ &= (-1)^{p+1}\chi\end{aligned} \tag{3.5}$$

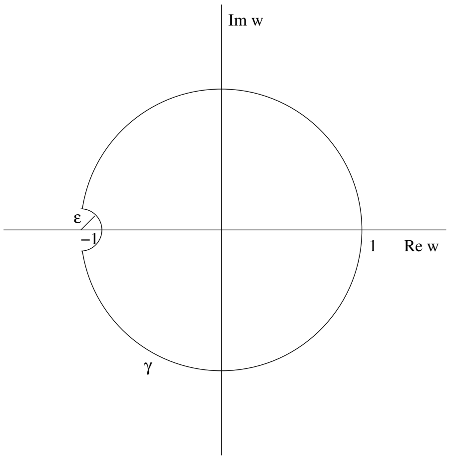

Figure 1: Regulated contour in the variable w.

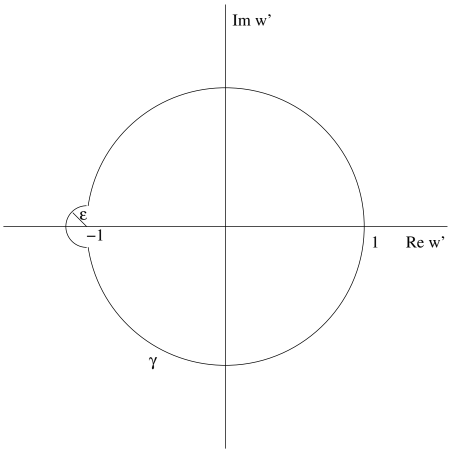

Figure 2: Regulated contour in the variable w'.

where χ is the Euler character of target space. In fact, the trace $\mathrm{Tr}\left[(-1)^{\hat{N}}\, e^{-\beta\hat{H}}\right]$ is seen to vanish, as the contributions from the F states are canceled by the contribution from the A states (recall the expansion (2.5)). Similarly, one can compute

$$\mathrm{Tr}\left[\hat{N}\,(-1)^{\hat{N}} e^{-\beta\hat{H}}\right] = -\chi\,. \tag{3.6}$$

Thus one obtains the following exact duality relation

$$Z_{D-p-1}(\beta) = Z_p(\beta) + (-1)^{p+1}\chi \tag{3.7}$$

where $Z_p(\beta)$ is defined in (3.4).

Let us compare this result with the one for massless tensors. The unregulated effective actions have a topological mismatch between a massless p-form and the dual massless $(D-p-2)$-form. It is related in even dimensions to the topological Euler character [10, 1], and in odd dimensions to the Ray-Singer torsion [11]. As we have just seen, a similar mismatch appears in even dimensions for a massive p-form and its dual massive $(D-p-1)$-form, and it is again proportional to the topological Euler character. It is worth noting that these mismatches are topological in nature, and are given by integrals of local expressions of the metric. They can be subtracted when the effective actions are renormalized, so that the renormalized effective actions can be defined to preserve duality, as pointed out in [12]. We will explicitly see this at the level of regulated 2-point functions discussed in the next section.

4. Two-point function

We now wish to use the $N = 2$ spinning particle to compute the contribution to the graviton self-energy of massive and massless antisymmetric tensors of arbitrary rank in arbitrary spacetime dimensions. We choose to work in the vielbein basis, i.e. we take the 1-loop effective action to be a functional of the vielbein $\bar{\Gamma}_p^{QFT}[e_{a\rho}] \equiv \Gamma_p^{QFT}[g_{\mu\nu}(e_{a\rho})]$. If desired, one can convert the final result to the metric basis using the appropriate change of

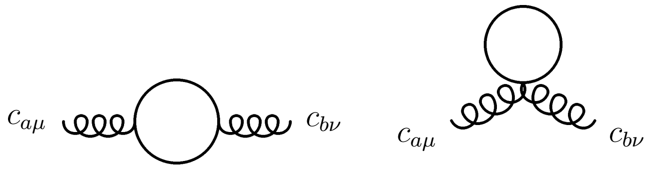

Figure 3: One-loop contributions to the graviton self-energy. External lines represents vielbein fluctuations.

variables (see eq. (4.14)). Alternatively, one can do the computation directly in the metric basis by rewriting the $N = 2$ spinning particle action in terms of worldline fermions with curved indices. In this setup, one should also remember to add measure ghosts for the fermions as well [9].

The two-point function in configuration space can be obtained from the 1-loop effective action by differentiating twice with respect to the vielbein (or metric) and then restricting to flat space. Alternatively, one can obtain the self-energy directly in momentum space by plugging into the worldline representation of the effective action, eq. (3.1), the following decomposition of the vielbein

$$\begin{aligned} e_{a\mu} &= \delta_{a\mu} + c_{a\mu} \;\rightarrow\; g_{\mu\nu} = \delta_{\mu\nu} + 2c_{(\mu\nu)} + c_{a\mu}c^a_\nu \\ c_{a\mu} &= \epsilon^{(1)}_{a\mu} e^{ik\cdot x(\tau)} + \epsilon^{(2)}_{a\mu} e^{-ik\cdot x(\tau)} \end{aligned} \tag{4.1}$$

and keeping only bilinear terms in the polarization tensors in the resulting calculation. In this way, one obtains the following expression for the two-point function

$$\bar\Gamma^{p-\mathrm{form}}_{(k,-k)}[e_{a\nu}] = -\frac{1}{2(2\pi)^{D/2}} \int_0^\infty \frac{d\beta}{\beta^{1+D/2}} e^{-\frac{1}{2}m^2\beta} \int_0^{2\pi} \frac{d\phi}{2\pi}\, \mu_{(m)}(\phi) \int d^D x_0 \big\langle e^{-S_{\mathrm{int}}} \big\rangle \Big|_{\mathrm{m.l.}} \tag{4.2}$$

where $\mu_{(m)}(\phi)$ is the appropriate measure for the U(1) modulus integration, i.e.

$$\mu_{(m)}(\phi) = \begin{cases} \left(2\cos\frac{\phi}{2}\right)^{D-2} e^{i(\frac{D}{2}-p-1)\phi} & m = 0 \\ \left(2\cos\frac{\phi}{2}\right)^{D-1} e^{i(\frac{D+1}{2}-p-1)\phi} & m \neq 0\,. \end{cases} \tag{4.3}$$

The correlation function $\langle e^{-S_{\mathrm{int}}}\rangle|_{\mathrm{m.l.}}$ (the suffix here indicates that we keep only terms which are multi-linear in the polarization tensors) is the same for massive and massless tensors and is computed from the non-linear sigma model action in eq. (3.3). Of course, the appropriate measure ghosts must also be included [13, 14].

To carry out the explicit calculation, we find it convenient to use the string-inspired propagators. The coordinates x are splitted as $x(\tau) = x_0 + y(\tau)$, where $y(\tau)$ satisfies periodic boundary conditions. The integration over the zero-mode x_0 gives momentum conservation, $(2\pi)^D\delta^D(\sum_i k_i)$, which we will not explicitly write in the following (and which we already used implicitly in eq. (4.1)). The detailed calculation of the correlation function in (4.2) is reported in the appendix. Here we quote the final result, which can be written as

$$\big\langle e^{-S_{\mathrm{int}}}\big\rangle\Big|_{\mathrm{m.l.}} = R_1 - \frac{1}{2}(R_2 - \bar R_2) + (A_0 - 1)(S_1 + S_2) + \frac{\beta k^2}{4}(2A_0 - 1)S_1 + \frac{\beta^2 k^4}{16} S_1 A_0 +$$

$$+\frac{1}{8\cos^2\frac{\phi}{2}}(S_2-2S_1)\left(\beta k^2(2A_0-1)+\frac{\beta^2k^4}{4}A_0\right), \tag{4.4}$$

where we defined the following set of tensors

$$\begin{aligned} R_1^{\mu\nu\alpha\beta} &= \delta^{\mu\nu}\delta^{\alpha\beta} \\ R_2^{\mu\nu\alpha\beta} &= \delta^{\mu\alpha}\delta^{\nu\beta}+\delta^{\mu\beta}\delta^{\nu\alpha} \\ \bar{R}_2^{\mu\nu\alpha\beta} &= \delta^{\mu\alpha}\delta^{\nu\beta}-\delta^{\mu\beta}\delta^{\nu\alpha}, \end{aligned} \tag{4.5}$$

along with the manifestly transverse tensors

$$\begin{aligned} S_1^{\mu\nu\alpha\beta} &= \left(\delta^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)\left(\delta^{\alpha\beta}-\frac{k^\alpha k^\beta}{k^2}\right) \\ S_2^{\mu\nu\alpha\beta} &= \left(\delta^{\mu\alpha}-\frac{k^\mu k^\alpha}{k^2}\right)\left(\delta^{\nu\beta}-\frac{k^\nu k^\beta}{k^2}\right)+(\mu\leftrightarrow\nu). \end{aligned} \tag{4.6}$$

In eq. (4.4) we have suppressed the polarization tensors by denoting $R_i=\epsilon^{(1)}_{\mu\nu}\epsilon^{(2)}_{\alpha\beta}R_i^{\mu\nu\alpha\beta}$ and $S_i=\epsilon^{(1)}_{\mu\nu}\epsilon^{(2)}_{\alpha\beta}S_i^{\mu\nu\alpha\beta}$, and we have also defined

$$A_0=\int_0^1 d\tau e^{-\frac{\beta}{2}k^2(\tau-\tau^2)}. \tag{4.7}$$

The first line of eq. (4.4) is just the contribution of a minimally coupled scalar particle running in the loop, as can be checked by comparing with the results of [8] [1], and the second line can be viewed as the correction due to "spin".

Inserting eq. (4.4) in eq. (4.2) and integrating over the moduli β and ϕ we get the following expression

$$\begin{aligned} \bar{\Gamma}^{p\text{-form}}_{(k,-k)} &= N_{dof}\bar{\Gamma}^{\text{scalar}}_{(k,-k)}-\frac{L_{(m)}(D,p)}{8(4\pi)^{\frac{D}{2}}}(S_2-2S_1)\times \\ &\times\left(k^2\Gamma\left(1-\frac{D}{2}\right)\left(2(K^2)^{\frac{D}{2}-1}-(m^2)^{\frac{D}{2}-1}\right)+\frac{k^4}{2}\Gamma\left(2-\frac{D}{2}\right)(K^2)^{\frac{D}{2}-2}\right) \end{aligned} \tag{4.8}$$

where we have defined

$$(K^2)^x=\int_0^1 d\tau\left(m^2+k^2\tau(1-\tau)\right)^x. \tag{4.9}$$

The number of propagating degrees of freedom N_{dof} is given by

$$N_{dof}=\int_0^{2\pi}\frac{d\phi}{2\pi}\mu_{(m)}(\phi)=\begin{cases}\dfrac{(D-2)!}{p!(D-2-p)!} & m=0\\ \dfrac{(D-1)!}{p!(D-1-p)!} & m\neq 0\end{cases} \tag{4.10}$$

[1] In that paper the computation was performed in the metric basis. One can however use the relation between correlation functions in vielbein and metric basis, see eq. (4.14).

and the coefficient $L_{(m)}(D,p)$ is equal to

$$L_{(m)}(D,p) = \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{\mu_{(m)}(\phi)}{\cos^2\frac{\phi}{2}} \tag{4.11}$$

or, in terms of the integrals $I_n(D,p)$ defined in paper I,

$$L_{(m)}(D,p) = \begin{cases} I_2(D,p) & m=0 \\ I_2(D+1,p) & m \neq 0\,. \end{cases} \tag{4.12}$$

All integrals over ϕ are regulated as prescribed in paper I (see also the previous section) by going to the Wilson loop variable $w = e^{i\phi}$ and deforming the contour in the complex plane to exclude the pole at $w=-1$. Note that $L_{(m)}(D,0)=0$ for any D, so for $p=0$ the result reduces to the scalar contribution $\bar\Gamma^{\text{scalar}}_{(k,-k)}$ as it should. Explicitly, this contribution is obtained from first line of eq. (4.4) after integrating over the proper time β and is given by

$$\begin{aligned}(4\pi)^{\frac{D}{2}}\bar\Gamma^{\text{scalar}}_{(k,-k)} = &-\frac{1}{2}\Gamma\Big(-\frac{D}{2}\Big)\Big((m^2)^{\frac{D}{2}}(R_1-\frac{1}{2}(R_2-\bar R_2))+((K^2)^{\frac{D}{2}}-(m^2)^{\frac{D}{2}})(S_1+S_2)\Big)-\\ &-\frac{1}{4}\Gamma\left(1-\frac{D}{2}\right)k^2\left(2(K^2)^{\frac{D}{2}-1}-(m^2)^{\frac{D}{2}-1}\right)S_1-\\ &-\frac{1}{8}\Gamma\left(2-\frac{D}{2}\right)k^4(K^2)^{\frac{D}{2}-2}S_1\,. \end{aligned}\tag{4.13}$$

Eq. (4.8) is our final result for the graviton self-energy in the vielbein basis. If one wishes to express the two-point function in the metric basis, one can use the following relation

$$\Gamma_{(k,-k)}[g_{\mu\nu}] = \frac{1}{4}\left(\bar\Gamma_{(k,-k)}[e_{a\nu}] - \epsilon^{(1)}_{\mu\nu}\epsilon^{(2)}_{\alpha\beta}\delta^{\mu\alpha}\bar\Gamma^{\nu\beta}_{(0)}[e_{a\nu}]\right) \tag{4.14}$$

which depends also on the one-point function $\bar\Gamma^{\nu\beta}_{(0)}[e_{a\nu}]$. This relation simply follows from $\bar\Gamma^{QFT}_p[e_{a\rho}] = \Gamma^{QFT}_p[g_{\mu\nu}(e_{a\rho})]$ and $g_{\mu\nu} = \delta_{ab}e^a{}_\mu e^b{}_\nu$. In the present case the one-point function is easily computed

$$\epsilon^{(1)}_{\mu\nu}\epsilon^{(2)}_{\alpha\beta}\bar\Gamma^{p-\text{form}\,\nu\beta}_{(0)}\delta^{\mu\alpha} = -\frac{N_{dof}}{4(4\pi)^{\frac{D}{2}}}(R_2+\bar R_2)(m^2)^{\frac{D}{2}}\Gamma\Big(-\frac{D}{2}\Big). \tag{4.15}$$

It only affects the scalar contribution in eq. (4.8), so the final expression for the two-point function in the metric basis is

$$\begin{aligned}\Gamma^{p-\text{form}}_{(k,-k)} &= N_{dof}\Gamma^{\text{scalar}}_{(k,-k)} - \frac{L_{(m)}(D,p)}{32(4\pi)^{\frac{D}{2}}}(S_2-2S_1)\times\\ &\times\left(k^2\Gamma\left(1-\frac{D}{2}\right)\left(2(K^2)^{\frac{D}{2}-1}-(m^2)^{\frac{D}{2}-1}\right)+\frac{k^4}{2}\Gamma\left(2-\frac{D}{2}\right)(K^2)^{\frac{D}{2}-2}\right)\end{aligned}\tag{4.16}$$

where $\Gamma^{\text{scalar}}_{(k,-k)}$ is the two-point function due to a minimally coupled scalar in the metric basis, which was given in [8] and is essentially the same as eq. (4.13) after dividing by 4 and replacing $\frac{1}{2}(R_2-\bar R_2)\to R_2$ (due to the tadpole correction in eq. (4.14)).

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As a consistency check, one can note that eq. (4.8) and eq. (4.16) satisfy the gravitational Ward identities (which can be found in [8]), since the scalar contribution was already checked to satisfy them in [8], and the additional correction is transversal and symmetric in each pair of indices (the one-point function contribution to the Ward identities only affects the scalar part).

We can now make several comments about the expected dualities of the final result for the two-point function. As noted in paper I for the massless case and in section 3 for the massive case, the mismatch between the (unregulated) effective actions of dual forms is due to the appearance of a pole at $\phi = \pi$, or $w = -1$. It was also shown that the mismatch is always related to topological invariants of the target space, namely the Euler character for even dimensions (massless and massive case) and the Ray-Singer torsion for odd dimensions (massless case only). Therefore one would expect that correlation functions, which are obtained differentiating the effective action with respect to the vielbein (or metric), do not show any mismatch even at the unrenormalized level. By looking at our final result, eq. (4.4) or eq. (4.16), and keeping in mind the measure factor for the integration over ϕ given in eq. (4.3), we see that the only possible poles at $\phi = \pi$ occur for $D = 2$ (both in massless and massive case) and $D = 3$ for massless forms. These poles affect the coefficient $L_{(m)}(D, p)$ which will not respect duality in those special cases. However, in $D = 2$ one has the identity $S_2 = 2S_1$, so the whole contribution proportional to $L_{(m)}(D, p)$ cancels out in this case. Moreover, one can verify that the expression in parenthesis in the second line of eq. (4.8) turns out to be proportional to $(D - 3)$ in the $m = 0$ limit, and thus the term multiplying $L_{(m)}(D, p)$ cancels out also in the $D = 3$ massless case. From these observations we may conclude that the two-point function, eq. (4.8), respects dualities in any dimension for both massive and massless forms. However, this is true formally at a given integer dimension D, but one has to be careful when applying dimensional regularization to regulate the proper time integral. In D odd there are no poles in DR, and indeed the two-point functions respect duality without any mismatch as one can infer from the above observations. For even D dimension, however, UV divergences arising from the proper time integral show up in DR as $1/\epsilon$ poles from the Γ-functions in eq. (4.8). But one is allowed to conclude that the mismatch between dual forms is topological only at a fixed integer D, while in DR ϵ-dependent terms which are not topological could appear. One can see such an effect by looking at the coefficients N_{dof} and $L_{(m)}(D, p)$. In fact, although these respect duality at a given integer D, they loose this property in DR. For example, taking a massless scalar and a massless 2-form in four dimensions:

$$\begin{aligned} N_{dof} &= 1 & p = 0 \quad D = 4 + \epsilon \\ N_{dof} &= 1 + \frac{3}{2}\epsilon + \frac{1}{2}\epsilon^2 & p = 2 \quad D = 4 + \epsilon\,. \end{aligned} \tag{4.17}$$

Thus, because of the $1/\epsilon$ pole from the Γ-function, dual forms receives different finite contributions to the two-point functions. However, the divergent $1/\epsilon$ part of the two-point function still respects strict duality as expected, since in this case one can keep everything at fixed integer D where the mismatch is topological and is not expected to appear in correlation functions. This finite discrepancy between dual forms is due to DR: in other schemes, such as a cutoff in proper time regularization, it should not appear. Of course, this finite mismatch will not affect the renormalized two-point function as it is local and one can always remove it in the renormalization procedure.

4.1 Massless p-form and the photon

Setting $m = 0$ in the final expression eq. (4.16), one gets the contribution to the graviton self-energy (in the metric basis) due to a massless p-form gauge potential

$$\Gamma^{p-\text{form}}_{(k,-k)}\Big|_{m=0} = N_{dof}\Gamma^{\text{scalar}}_{(k,-k)}\Big|_{m=0} - \frac{D-3}{D-1}L_{(0)}(D,p)\frac{(k^2)^{\frac{D}{2}}}{64(4\pi)^{\frac{D}{2}}} \times \\ \times\Gamma\left(2-\frac{D}{2}\right)B\left(\frac{D}{2}-1,\frac{D}{2}-1\right)(S_2-2S_1) \tag{4.18}$$

where $B(x,y) = \Gamma(x)\Gamma(x)/\Gamma(x+y)$ is the Euler beta function. The contribution of the massless scalar can be written as

$$\Gamma^{\text{scalar}}_{(k,-k)}\Big|_{m=0} = -\frac{(k^2)^{\frac{D}{2}}}{32(4\pi)^{\frac{D}{2}}}\Gamma\left(2-\frac{D}{2}\right)B\left(\frac{D}{2}-1,\frac{D}{2}-1\right)\left[\frac{D^2-2D-2}{D^2-1}S_1+\frac{1}{D^2-1}S_2\right]. \tag{4.19}$$

The graviton self-energy due to a photon loop was previously computed in [2] using standard QFT techniques. To compare with their result, one can set $p = 1$ in the above general expression for massless tensors. In this case $N_{dof} = D-2$ and $L_{(0)}(D,1) = 4$, so that we get

$$\Gamma^{\text{photon}}_{(k,-k)} = -\frac{(k^2)^{\frac{D}{2}}}{32(4\pi)^{\frac{D}{2}}}\Gamma\left(2-\frac{D}{2}\right)B\left(\frac{D}{2}-1,\frac{D}{2}-1\right)\times \\ \times\left[\frac{D^3-8D^2+10D+16}{D^2-1}S_1+\frac{2D^2-3D-8}{D^2-1}S_2\right]. \tag{4.20}$$

One can check that this result agrees with the one in [2]. To make the comparison one has to keep in mind that in [2] correlation functions are defined by fluctuations of $\hat{g}^{\mu\nu} \equiv \sqrt{g}g^{\mu\nu}$, while we consider fluctuations of the metric $g_{\mu\nu}$.

5. Stückelberg fields for massive tensors

In this section we will show how the $N = 2$ worldline description of massive tensors naturally suggests a way of rewriting of the QFT Proca action in terms of Stückelberg fields and an abelian gauge invariance with the relative ghosts for ghosts tower.

We first review the case of massless tensors. As discussed in detail in paper I, the massless $N = 2$ spinning particle describes a generalized field strength F_{p+1} satisfying Maxwell equations and Bianchi identities. This constraint is enforced by gauging the $N = 2$ worldline supersymmetry. The net effect of this gauging is to produce a factor $(2\cos\frac{\phi}{2})^{-2} = \frac{w}{(1+w)^2}$ coming from the susy ghosts which modifies the measure for the U(1) modular integration. In fact, one can see that this factor coming from worldline susy ghosts automatically encodes the appropriate ghosts for ghosts tower needed to covariantly gauge fix the gauge potential A_p. This is most easily seen by considering the operatorial representation of the partition function for the p-form gauge potential

$$Z_p(\beta) = \oint_\gamma \frac{dw}{2\pi i w}\frac{w}{(1+w)^2}\,\mathrm{Tr}\left[w^{\hat{N}-p-1}e^{-\beta\hat{H}}\right] = \oint_\gamma \frac{dw}{2\pi i w}\frac{w}{(1+w)^2}\sum_{n=0}^{D} w^{n-p-1}t_n(\beta) \quad (5.1)$$

where $t_n(\beta)$ is $\mathrm{Tr}\, e^{-\beta\hat{H}}$ restricted to the space of n-forms (the sector of the Hilbert space with occupation number $\hat{N}=n$). The pole at $w=-1$ is excluded, thus we only need to consider the pole at $w=0$. Around $w=0$, one has the expansion

$$\frac{1}{(1+w)^2} = 1 - 2w^2 + 3w^3 - 4w^4 + \cdots \quad (5.2)$$

whose coefficients are seen to give the right multiplicity and statistics of the ghosts for ghosts tower. Indeed, plugging this expansion into eq. (5.1) and using Cauchy theorem one gets

$$Z_p(\beta) = \sum_{n=0}^{p}(-1)^n(n+1)t_{p-n}(\beta) \quad (5.3)$$

which corresponds to the correct triangular ghost structure for the p-form gauge potential.

We now turn to the massive case. As explained in section 2, gauging supersymmetry gives the Proca field equations for a massive tensor A_p. The worldline susy ghost factor now combines with the determinant arising by integrating out the fermionic θ variables, to give a net factor $(2\cos\frac{\phi}{2})^{-1}$ in the measure. Following the lines of the massless case, we expect that this factor could be viewed to create some kind of "auxiliary field" tower for the massive Proca tensor. In fact, consider again the operatorial form of the partition function

$$\begin{aligned} Z_p(\beta) &= \oint_\gamma \frac{dw}{2\pi i w}\frac{w}{(1+w)^2}\,\mathrm{Tr}\left[w^{\hat{\psi}^\mu\hat{\psi}^\dagger_\mu+\hat{\theta}\hat{\theta}^\dagger-p-1}e^{-\beta\hat{H}}\right] \\ &= \oint_\gamma \frac{dw}{2\pi i w}\frac{w}{1+w}\,\mathrm{Tr}'\left[w^{\hat{\psi}^\mu\hat{\psi}^\dagger_\mu-p-1}e^{-\beta\hat{H}}\right] \\ &= \oint_\gamma \frac{dw}{2\pi i w}\frac{w}{1+w}\sum_{n=0}^{D} w^{n-p-1}\,t_n(\beta) \end{aligned} \quad (5.4)$$

where first we have traced over eigenstates of $\hat{\theta}\hat{\theta}^\dagger$ to produce a factor $(1+w)$ (note that $\hat{H}$ does not depend on $\hat{\theta}$'s), and then expressed the remaining trace (Tr') over eigenstates of $\hat{\psi}^\mu\hat{\psi}^\dagger_\mu$, i.e. the n-forms of the restricted Hilbert space. Now the measure factor has the following expansion around $w=0$

$$\frac{1}{1+w} = 1 - w^2 + w^3 - w^4 + \cdots \quad (5.5)$$

Plugging into eq. (5.4) we get

$$\begin{aligned} Z_p(\beta) &= t_p(\beta) - t_{p-1}(\beta) + t_{p-2}(\beta) - \cdots \\ &= \sum_{n=0}^{p}(-1)^n t_{p-n}(\beta)\,. \end{aligned} \quad (5.6)$$

This suggests that the Proca action could be rewritten in terms of a tower of differential forms with simple kinetic operator (the generalized laplacian $dd^\dagger + d^\dagger d$). One can check that the above relation is correct by comparing the Seeley-DeWitt coefficients for massive forms given in appendix with the coefficients for differential forms with generalized laplacian as kinetic operator which were computed in paper I. Also notice that eq. (5.6) is in agreement with the exact duality relations derived in section 3.

We now would like to interpret eq. (5.6) from the QFT point of view. This will produce a useful way of rewriting the Proca action. For simplicity, consider $p = 1$ (massive photon) and flat space. The generalization to curved space is straightforward. The Proca action is

$$S^{QFT}[A_\mu] = \int d^D x \left[-\frac{1}{4} F_{\mu\nu}^2 - \frac{m^2}{2} A_\mu^2 \right] . \tag{5.7}$$

This action does not have any gauge invariance. However, we could add by hand a scalar field ϕ (a Stückelberg field) in the following way

$$S^{QFT}[A_\mu, \phi] = \int d^D x \left[-\frac{1}{4} F_{\mu\nu}^2 - \frac{m^2}{2} \left(A_\mu - \frac{1}{m} \partial_\mu \phi \right)^2 \right] . \tag{5.8}$$

Clearly now we have the gauge invariance

$$\begin{aligned} \delta A_\mu &= \partial_\mu \Lambda \\ \delta \phi &= m \Lambda \, . \end{aligned} \tag{5.9}$$

Of course, ϕ could be completely gauged away from the action by using this symmetry. To our purpose it is convenient to keep it and choose a different gauge fixing. Following the standard BRST procedure, we introduce the ghost c, the non-minimal fields (b, π), and the BRST transformations

$$\begin{aligned} \delta_B A_\mu &= \partial_\mu c \\ \delta_B \phi &= mc \\ \delta_B c &= 0 \\ \delta_B b &= \pi \\ \delta_B \pi &= 0 \, . \end{aligned} \tag{5.10}$$

Choosing the following gauge fermion

$$\Psi = b \left(\partial_\mu A^\mu - m\phi + \frac{1}{2} \pi \right) , \tag{5.11}$$

the gauge fixed action gets the form

$$\begin{aligned} S^{QFT}[A_\mu, \phi, b, c] &= \int d^D x \left[-\frac{1}{4} F_{\mu\nu}^2 - \frac{m^2}{2} (A_\mu - \frac{1}{m} \partial_\mu \phi)^2 + \delta_B \Psi \right] \\ &= \int d^D x \left[\frac{1}{2} A_\mu (\partial^2 - m^2) A^\mu + \frac{1}{2} \phi (\partial^2 - m^2) \phi - b (\partial^2 - m^2) c \right] \end{aligned} \tag{5.12}$$

where in the last line the auxiliary field π was integrated out. This shows that the massive photon is equivalent to the sum of a vector, a commuting scalar (ϕ) and two anticommuting scalars (b, c), all with the simplest kinetic operator. It is easy to see that the alternating tower of eq. (5.6) is reproduced. One advantage of using these extra fields is to have simpler kinetic terms, and thus simpler propagators. A description of this Stückelberg field approach to massive spin 1 fields can also be found in the textbook [15].

It is straightforward to generalize this procedure for tensors of higher rank. For example, for the massive 2-form A_2 one adds a Stückelberg vector field ϕ_1 in the Proca action through the substitution

$$A_2 \to A_2 - \frac{1}{m} d\phi_1 \,. \tag{5.13}$$

This introduces the gauge symmetries $\delta A_2 = d\Lambda_1$ and $\delta\phi_1 = m\Lambda_1 + d\Lambda_0$. Upon gauge fixing one gets two anticommuting vector ghosts and three commuting scalar ghosts from the Λ_1-symmetry, plus two anticommuting scalar ghosts from the Λ_0-symmetry. Therefore the alternating structure of eq. (5.6) is again reproduced.

6. Conclusions

We have extended the worldline description of massless antisymmetric tensor fields coupled to gravity initiated in paper I to the massive case, and used the worldline description to study various quantum effects. In particular, we have computed the one-loop effective action in its proper time expansion (a local derivative expansion) and obtained the first few Seeley-DeWitt coefficients (a_0, a_1, a_2). This derivative expansion is really valid only for massive fields, because otherwise infrared divergences appear. However the Seeley-DeWitt coefficients are useful in the massless case as well: they identify, for example, the counterterms needed to renormalize the full effective action. Then we have studied exact duality relations between antisymmetric tensor fields and obtained the topological terms that cause a mismatch in their unregulated effective action. Finally we have computed the two-point function of the metric with an arbitrary rank p antisymmetric tensor field in the loop, either massive or massless. For the particular case of a massless 1-form, we have reproduced the result obtained in [2]. The worldline perspective employed here and in paper I has been instrumental to obtain general results in the quantum theory of antisymmetric tensor fields, which may find some applications in theoretical particle and string physics.

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A. List of Seeley-DeWitt coefficients for massive p-forms A_p

One may write the one-loop effective action for a massive p-form A_p as follows

$$\begin{aligned}\Gamma_p^{QFT}[g_{\mu\nu}] &= -\frac{1}{2}\int_0^\infty \frac{d\beta}{\beta} e^{-\frac{1}{2}m^2\beta}\, Z_p(\beta) \\ Z_p(\beta) &= \int \frac{d^D x\sqrt{g}}{(2\pi\beta)^{\frac{D}{2}}}\big(a_0 + a_1\beta + a_2\beta^2 + \cdots\big)\end{aligned} \tag{A.1}$$

where the coefficients a_i are the so-called Seeley-DeWitt coefficients in the coincidence limit. We parametrize them as follows

$$\big(v_1 + v_2 R\beta + (v_3 R^2_{abcd} + v_4 R^2_{ab} + v_5 R^2 + v_6\nabla^2 R)\beta^2 + \cdots\big)\,. \tag{A.2}$$

Then using the format

$$A_p \to (v_1; v_2; v_3; v_4; v_5; v_6) \tag{A.3}$$

we may list their explicit values for $p = 0, 1, 2, 3, 4, 5$

$$A_0 \to \left(1; \frac{1}{12}; \frac{1}{720}; -\frac{1}{720}; \frac{1}{288}; \frac{1}{120}\right)$$

$$A_1 \to \left(D-1; \frac{D-7}{12}; \frac{D-16}{720}; \frac{91-D}{720}; \frac{D-13}{288}; \frac{D-6}{120}\right)$$

$$\begin{aligned}A_2 \to \bigg(&\frac{(D-1)(D-2)}{2}; \frac{D^2-15D+38}{24}; \frac{(D-16)(D-17)}{1440}; \\ &-\frac{D^2-183D+1262}{1440}; \frac{D^2-27D+146}{576}; \frac{D^2-13D+32}{240}\bigg)\end{aligned}$$

$$\begin{aligned}A_3 \to \bigg(&\frac{1}{6}(D-1)(D-2)(D-3); \frac{1}{72}(D-3)(D^2-21D+74); \\ &\frac{1}{4320}(D^3-51D^2+866D-3246); -\frac{1}{4320}(D^3-276D^2+4061D-14046); \\ &\frac{1}{1728}(D-11)(D^2-31D+138); \frac{1}{720}(D-3)(D^2-18D+62)\bigg)\end{aligned}$$

$$\begin{aligned}A_4 \to \bigg(&\frac{1}{24}(D-1)(D-2)(D-3)(D-4); \frac{1}{288}(D-3)(D-4)(D^2-27D+122); \\ &\frac{1}{17280}(D^4-70D^3+1835D^2-14750D+36024); \\ &-\frac{1}{17280}(D^4-370D^3+8675D^2-64490D+151224); \\ &\frac{1}{6912}(D^4-58D^3+1043D^2-7058D+15864); \\ &\frac{1}{2880}(D-3)(D-4)(D-6)(D-17)\bigg)\end{aligned}$$

$$A_5 \to \bigg(\frac{1}{120}(D-1)(D-2)(D-3)(D-4)(D-5);$$

$$
\begin{aligned}
&\frac{1}{1440}(D-3)(D-4)(D-5)(D-7)(D-26);\\
&\frac{1}{86400}(D-5)(D^4-85D^3+2810D^2-27500D+81024);\\
&-\frac{1}{86400}(D-5)(D^4-460D^3+13085D^2-117950D+334824);\\
&\frac{1}{34560}(D-5)(D^4-70D^3+1535D^2-12650D+34584);\\
&\left.\frac{1}{14400}(D-3)(D-4)(D-5)(D^2-28D+152)\right). \qquad \text{(A.4)}
\end{aligned}
$$

They all satisfy the duality relations discussed in the text.

B. Computation of the two-point function

The relevant contributions to the correlation function $\langle e^{-S_{\rm int}}\rangle|_{\rm m.l.}$ appearing in eq. (4.2) are the following

$$
\begin{aligned}
\langle e^{-S_{\rm int}}\rangle\Big|_{\rm m.l.} = &-\frac{1}{2\beta}\int_0^1 d\tau\langle\, c_{a\mu}c^a_\nu(\dot y^\mu \dot y^\nu + a^\mu a^\nu + b^\mu c^\nu)\rangle - &\text{(A)}\\
&-\frac{1}{\beta}\int_0^1 d\tau\langle \dot y^\mu \omega^{(2)}_{\mu ab}\bar\psi^a\psi^b\rangle + &\text{(B)}\\
&+\frac{1}{2\beta}\int_0^1 d\tau\langle\, R^{(2)}_{abcd}\bar\psi^a\psi^b\bar\psi^c\psi^d\rangle + &\text{(C)}\\
&+\frac{1}{8\beta^2}\left\langle\left(\int_0^1 d\tau 2c_{(\mu\nu)}(\dot y^\mu \dot y^\nu + a^\mu a^\nu + b^\mu c^\nu)\right)^2\right\rangle + &\text{(D)}\\
&+\frac{1}{2\beta^2}\left\langle\left(\int_0^1 d\tau \dot y^\mu \omega^{(1)}_{\mu ab}\bar\psi^a\psi^b\right)^2\right\rangle + &\text{(E)}\\
&+\frac{1}{2\beta^2}\left\langle \int_0^1 d\tau\int_0^1 d\sigma 2c_{(\mu\nu)}(\dot y^\mu \dot y^\nu + a^\mu a^\nu + b^\mu c^\nu)(\tau)\dot y^\mu\omega^{(1)}_{\mu ab}\bar\psi^a\psi^b(\sigma)\right\rangle + &\text{(F)}\\
&+\frac{1}{8\beta^2}\left\langle\left(\int_0^1 d\tau R^{(1)}_{abcd}\bar\psi^a\psi^b\bar\psi^c\psi^d\right)^2\right\rangle - &\text{(G)}\\
&-\frac{1}{4\beta^2}\left\langle \int_0^1 d\tau\int_0^1 d\sigma 2c_{(\mu\nu)}(\dot y^\mu \dot y^\nu + a^\mu a^\nu + b^\mu c^\nu)(\tau)R^{(1)}_{abcd}\bar\psi^a\psi^b\bar\psi^c\psi^d(\sigma)\right\rangle - &\\
& &\text{(H)}\\
&-\frac{1}{2\beta^2}\left\langle \int_0^1 d\tau\int_0^1 d\sigma \dot y^\mu\omega^{(1)}_{\mu ab}\bar\psi^a\psi^b(\tau)R^{(1)}_{cdef}\bar\psi^c\psi^d\bar\psi^e\psi^f(\sigma)\right\rangle &\text{(I)}
\end{aligned}
\qquad \text{(B.1)}
$$

where $R^{(i)}_{abcd}$ and $\omega^{(i)}_{\mu ab}$ denote the part of the Riemann tensor and spin connection with i powers of the vielbein fluctuation $c_{a\mu}$. The first three contributions in eq. (B.1) correspond to one-vertex graphs, while the remaining six contributions correspond to two-vertex graphs (see figure 3). The calculation is a straightforward generalization of the analogous ones performed in [8] and [9] for the scalar and the fermion. It is convenient to use the following string inspired propagators

$$
\begin{aligned}
\langle y^\mu(\tau) y^\nu(\sigma)\rangle &= -\beta\delta^{\mu\nu}\Delta(\tau-\sigma) \\
\langle a^\mu(\tau) a^\nu(\sigma)\rangle &= \beta\delta^{\mu\nu}\Delta_{gh}(\tau-\sigma) \\
\langle b^\mu(\tau) c^\nu(\sigma)\rangle &= -2\beta\delta^{\mu\nu}\Delta_{gh}(\tau-\sigma) \\
\langle \psi^a(\tau) \bar{\psi}_b(\sigma)\rangle &= \beta\delta^a_b\Delta_{AF}(\tau-\sigma)
\end{aligned} \tag{B.2}
$$

where Δ, Δ_{gh} and Δ_{AF} are given by

$$
\begin{aligned}
\Delta(x) &= -\sum_{n\neq 0}\frac{1}{4\pi^2 n^2}e^{2\pi i n x} = \frac{1}{2}|x| - \frac{1}{2}x^2 - \frac{1}{12} \\
\Delta_{gh}(x) &= \sum_{n=-\infty}^{\infty} e^{2\pi i n x} = \delta(x) \\
\Delta_{AF}(x) &= \sum_{r\in Z+\frac{1}{2}} \frac{-i}{2\pi r+\phi}\, e^{2\pi i r x} = \frac{e^{-i\phi x}}{2\cos\frac{\phi}{2}}\left[e^{i\frac{\phi}{2}}\theta(x) - e^{-i\frac{\phi}{2}}\theta(-x)\right],
\end{aligned} \tag{B.3}
$$

and dimensional regularization as described in paper I.

To organize the computation, it is convenient to introduce the following basis of tensors

$$
\begin{aligned}
R_1^{\mu\nu\alpha\beta} &= \delta^{\mu\nu}\delta^{\alpha\beta} \\
R_2^{\mu\nu\alpha\beta} &= \delta^{\mu\alpha}\delta^{\nu\beta} + \delta^{\mu\beta}\delta^{\nu\alpha} \\
\bar{R}_2^{\mu\nu\alpha\beta} &= \delta^{\mu\alpha}\delta^{\nu\beta} - \delta^{\mu\beta}\delta^{\nu\alpha} \\
R_3^{\mu\nu\alpha\beta} &= \frac{1}{k^2}\left(\delta^{\mu\alpha}k^\nu k^\beta + \delta^{\nu\alpha}k^\mu k^\beta + \delta^{\mu\beta}k^\nu k^\alpha + \delta^{\nu\beta}k^\mu k^\alpha\right) \\
R_4^{\mu\nu\alpha\beta} &= \frac{1}{k^2}\left(\delta^{\mu\nu}k^\alpha k^\beta + \delta^{\alpha\beta}k^\mu k^\nu\right) \\
R_5^{\mu\nu\alpha\beta} &= \frac{1}{k^4}\, k^\mu k^\nu k^\alpha k^\beta\,.
\end{aligned} \tag{B.4}
$$

It will be also useful to define the following transverse combinations

$$
\begin{aligned}
S_1^{\mu\nu\alpha\beta} &= (R_1 - R_4 + R_5)^{\mu\nu\alpha\beta} \\
S_2^{\mu\nu\alpha\beta} &= (R_2 - R_3 + 2R_5)^{\mu\nu\alpha\beta}\,.
\end{aligned} \tag{B.5}
$$

In the following, we will suppress the explicit polarization tensors by use of the notation $R_i = \epsilon^{(1)}_{\mu\nu}\epsilon^{(2)}_{\alpha\beta}R_i^{\mu\nu\alpha\beta}$. Explicitly, we get the following results:

- (A) was already computed for the $N=1$ sigma model [9], and is equal to

$$
(A) = \frac{1}{2}(R_2 + \bar{R}_2)\,. \tag{B.6}
$$

- (B) vanishes being proportional to $\omega_{\mu ab}\delta^{ab}$.
- (C) is given by

$$
(C) = \frac{1}{2\beta}\int_0^1 d\tau \langle\, R^{(2)}_{abcd}\bar{\psi}^a\psi^b\bar{\psi}^c\psi^d\,\rangle = \frac{\beta}{2}\Delta^2_{AF}(0)\int_0^1 d\tau\langle\, R^{(2)}\,\rangle\,. \tag{B.7}
$$

The Ricci-scalar correlation function has already been computed for the scalar field in the $N = 0$ sigma model [8], using the metric basis. The result in the vielbein basis is four times that result

$$\langle R^{(2)} \rangle = -k^2 (2R_1 + R_2 - R_3)\,. \tag{B.8}$$

So we obtain

$$(C) = \frac{\beta k^2}{8} \tan^2 \frac{\phi}{2} (2R_1 + R_2 - R_3)\,. \tag{B.9}$$

- (D) has also been computed already for the scalar [8] and the fermion [9] running in the loop. The result in the vielbein basis is just four times the result in the metric basis, which was presented in detail in [8]

$$\begin{aligned}(D) &= \frac{1}{8\beta^2} \left\langle \left(\int_0^1 d\tau 2c_{(\mu\nu)} (\dot{y}^\mu \dot{y}^\nu + a^\mu a^\nu + b^\mu c^\nu) \right)^2 \right\rangle \\ &= A_0 (S_1 + S_2) + \frac{\beta k^2}{8} S_2 + (R_3 + R_4 - 2R_2 - 3R_5) \end{aligned} \tag{B.10}$$

where

$$A_0 = \int_0^1 d\tau e^{-\frac{\beta}{2} k^2 (\tau - \tau^2)}\,. \tag{B.11}$$

- The computation of (E) is very similar to the computation of the analogous term in the $N = 1$ case. It needs dimensional regularization (DR) and the final result turns out to be

$$(E) = \frac{1}{2\beta^2} \left\langle \left(\int_0^1 d\tau \dot{y}^\mu \omega^{(1)}_{\mu ab} \bar{\psi}^a \psi^b \right)^2 \right\rangle = \frac{\beta k^2}{4\cos^2 \frac{\phi}{2}} S_2 (A_0 - 1)\,. \tag{B.12}$$

- (F) vanishes for the same reason as (B).
- (G) can be computed by first contracting all the fermions

$$\begin{aligned}(G) &= \frac{1}{8\beta^2} \left\langle \left(\int_0^1 d\tau R^{(1)}_{abcd} \bar{\psi}^a \psi^b \bar{\psi}^c \psi^d \right)^2 \right\rangle \\ &= \frac{\beta^2}{8} \int_0^1 d\tau \int_0^1 d\sigma \Big(\Delta^4_{AF}(0) \langle R^{(1)}(\tau) R^{(1)}(\sigma) \rangle - \\ &\qquad - 4\Delta^2_{AF}(0) \Delta_{AF}(\tau - \sigma) \Delta_{AF}(\sigma - \tau) \langle R^{(1)}_{ab}(\tau) R^{(1)}_{ab}(\sigma) \rangle + \\ &\qquad + \Delta^2_{AF}(\tau - \sigma) \Delta^2_{AF}(\sigma - \tau) \langle R^{(1)}_{abcd}(\tau) R^{(1)}_{abcd}(\sigma) \rangle \Big)\,. \end{aligned} \tag{B.13}$$

The remaining correlation functions yield

$$\begin{aligned}\langle R^{(1)}(\tau) R^{(1)}(\sigma) \rangle &= 8k^4 S_1 e^{-\beta k^2 \Delta_0 (\tau - \sigma)} \\ \langle R^{(1)}_{ab}(\tau) R^{(1)}_{ab}(\sigma) \rangle &= k^4 (2S_1 + S_2) e^{-\beta k^2 \Delta_0 (\tau - \sigma)}\end{aligned}$$

$$\langle R^{(1)}_{abcd}(\tau) R^{(1)}_{abcd}(\sigma)\rangle = 4k^4 S_2 e^{-\beta k^2 \Delta_0(\tau-\sigma)}\,, \tag{B.14}$$

where $\Delta_0(\tau-\sigma) = \Delta(\tau-\sigma) - \Delta(0)$. The resulting worldline integrals do not need DR, and the final result is

$$\begin{aligned}(G) &= \frac{\beta^2 k^4}{16}\left(\tan^4\frac{\phi}{2} S_1 - \frac{\tan^2\frac{\phi}{2}}{2\cos^2\frac{\phi}{2}}(2S_1+S_2) + \frac{1}{2\cos^4\frac{\phi}{2}} S_2\right) A_0 \\ &= \frac{\beta^2 k^4}{16}\left(S_1 + \frac{1}{2\cos^2\frac{\phi}{2}}(S_2 - 2S_1)\right) A_0\,. \end{aligned} \tag{B.15}$$

Note that the terms proportional to $\cos^{-4}\frac{\phi}{2}$ cancel out identically. This is going to be important for the expected duality of the final result.

- After contracting the four fermions, (H) becomes

$$(H) = -\frac{1}{4}\int_0^1 d\tau \int_0^1 d\sigma \langle 2c_{(\mu\nu)}(\dot{y}^\mu \dot{y}^\nu + a^\mu a^\nu + b^\mu c^\nu)(\tau) R^{(1)}\rangle \Delta^2_{AF}(0)\,. \tag{B.16}$$

The remaining correlation function was already computed for the scalar in the $N=0$ model, the result in the vielbein basis is 4 times that result, and we get

$$(H) = -\frac{\beta k^2}{2}\tan^2\frac{\phi}{2}\left(A_0 S_1 - R_5 + \frac{1}{2}R_4\right)\,. \tag{B.17}$$

- Finally, (I) vanishes being proportional to $\omega_{\mu ab}R_{ab}$.

Summing up all the contributions in eq. (B.1), we get the expression in eq. (4.4).

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